

An alternative approach to dynamical mass generation in QED_3

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Abstract

Some quantum properties of QED_3 are studied with the help of an exact evolution equation of the effective action with the bare fermion mass. The resulting effective theory and the occurrence of a dynamical mass are discussed in the framework of the gradient expansion.

1 Introduction

QED in 2+1 dimensions (QED_3) was introduced as a toy model for chiral symmetry breaking [1, 2]. There the Schwinger-Dyson equation for the fermion propagator was solved, in order to study the dynamical fermion mass generation. More recently, similar studies were done analytically and numerically [3], where different resummations are considered in the study of the Schwinger-Dyson equation, taking into account different ansätze for the the vertex.

Besides the toy model motivation, QED_3 happens to be relevant for the effective description of planar antiferromagnetic systems [4]. Indeed, the d -wave pairing that occurs in these systems allows a linearization of the dispersion relation around the nodes in the energy gap, which leads to relativistic-like effective theories.

Motivated by these recent developments, an alternative approach to the quantum properties of QED_3 is proposed here. This approach is based on an exact functional method, where the quantum fluctuations are controlled via the bare fermion mass. When this mass is large, the interaction can be neglected and the quantum corrections are weak: the system is almost classical. As the bare mass decreases, the quantum fluctuations gradually appear and the system sees the generation of quantum effects, leading to the full quantum effective theory.

This functional method was also used in 3+1 dimensions for a scalar theory [5] and for QED [6], in the description of a coulomb gas [7] and in a Wess-Zumino model in 2+1 dimensions [8]. In all these cases, the usual renormalized parameters were recovered at one-loop. Beyond one-loop, the resummation provided by the gradient expansion used in this functional method does not coincide anymore with the loop expansion. In the example of QED and for the one-loop approximation, a singularity appears in the coupling constant flow as the bare fermion mass decreases, i.e. as the fluctuations appear. This singularity is the analogue of the famous Landau pole and it disappears beyond the one-loop approximation, i.e. when taking into account the full equations obtained in this method.

In section 2, we set up the frame of this method and give the resulting flows that are obtained, the computations being given in Appendix A and Appendix B. We show then in section 3 that the resulting effective couplings are consistent, at one-loop, with the usual renormalized parameters computed with standard Feynman graphs. In section 4, we give the non-perturbative solution and discuss the generation of a dynamical mass in the massless case ("massless" refers here to the bare theory). A self-consistent relation is found between the effective coupling and the dynamically generated mass. This relation shows that the only way to have a vanishing dynamical mass is to have a non-interacting effective theory. This result is set in the lowest order of the gradient expansion and thus the influence of the number of flavours is not discussed, as explained in section 5 which is left for a discussion.

2 Evolution equations

We consider a parity conserving theory, what is possible with $2N$ fermions, constituting N couples in which the masses are opposite. The $2N$ fermions are then written as N 4-component fermions and we consider the 4×4 reducible representation of the γ -matrices given in [2].

The starting point is the bare action

$$S = \int d^3x \left\{ \frac{1}{2} A^\mu (T_{\mu\nu} + \alpha L_{\mu\nu}) A^\nu \right. \quad (1)$$

$$+ \sum_{k=1}^N \bar{\psi}_k (i \not{\partial} - g_B \not{A} - \lambda m_B) \psi_k \},$$

where $T_{\mu\nu}$ and $L_{\mu\nu}$ are respectively the transverse and longitudinal parts of the photon propagator and α is a usual gauge-fixing parameter.

The control of the quantum effects consists in playing with λ . As this parameter goes to ∞ , the fermion bare mass is large and thus freezes the quantum fluctuations: the system is classical and the effective theory is the bare one. On the other hand, as λ decreases, the quantum corrections gradually appear in the effective theory.

The idea of the method is to compute the evolution of the effective action Γ (the proper graphs generator functional) with λ . Starting from $\lambda = \infty$, where $\Gamma = S$, the massive case will be obtained when λ decreases down to 1 (see section 3) and the massless case when λ decreases down to 0 (see section 4).

The evolution equation for Γ was derived for $N = 1$ in [6] and we review in the Appendix A this derivation for any N . The result reads

$$\partial_\lambda \Gamma = m_B \text{Tr} \sum_{k=1}^N \left\{ -\bar{\psi}_k \psi_k + i \left(\frac{\delta^2 \Gamma}{\delta \bar{\psi}_k \delta \psi_k} \right)^{-1} \right\}, \quad (2)$$

and we recall that this evolution equation is exact and does not rely on any approximation. In the framework of the gradient expansion, we then make the projection of Γ on the following functional subspace:

$$\begin{aligned} \Gamma = & \int d^3x \left\{ \frac{1}{2} A^\mu (\beta_T T_{\mu\nu} + \alpha \beta_L L_{\mu\nu}) A^\nu \right. \\ & \left. + \sum_{k=1}^N \bar{\psi}_k (i z \not{\partial} - g z \not{A} - m) \psi_k \right\}, \end{aligned} \quad (3)$$

where $\beta_T, \beta_L, z, g, m$ are functions of λ . The step (3) constitutes our approximation and is valid in the IR. It is shown in the Appendix B that plugging this ansatz in the evolution equation (2) leads to the following differential equations:

$$\begin{aligned} \partial_\lambda \beta_L &= 0 \\ \partial_\lambda m &= m_B \\ \partial_\lambda \beta_T &= -N \frac{g^2 z m_B}{6\pi m^2} \\ \partial_\lambda z &= \frac{g^2 z^2 m_B}{6\pi m^2} \frac{1}{\alpha \beta_L} \\ \partial_\lambda (gz) &= \frac{g^3 z^2 m_B}{18\pi m^2} \left(\frac{2}{\beta_T} + \frac{1}{\alpha \beta_L} \right) \end{aligned} \quad (4)$$

It is interesting to note the following points:

- As expected, the photon does not acquire a mass in the evolution with λ .
- The longitudinal part β_L of the photon propagator does not evolve with λ .
- In the Landau gauge ($\alpha = \infty$), there is no fermion wave function renormalization and $z = 1$.
- The fermion mass m has a classical flow with λ (no quantum contribution).

As a consequence, we have

$$\begin{aligned}\beta_L &= 1, \\ m &= m_B(\lambda + c),\end{aligned}\tag{5}$$

where c is a constant of integration linked to the dynamical mass generated in the massless case since.

$$m_{dyn} = \lim_{\lambda \rightarrow 0} \left(\frac{m}{z} \right) = c \frac{m_B}{z_0},\tag{6}$$

where $z_0 = z(0)$.

If we note $\beta_T = \beta$, the remaining differential equations can be written

$$\begin{aligned}\partial_\lambda \beta &= -N \frac{g^2 z}{6\pi m_B} \frac{1}{(\lambda + c)^2} \\ \partial_\lambda z &= \frac{1}{\alpha} \frac{g^2 z^2}{6\pi m_B} \frac{1}{(\lambda + c)^2} \\ \partial_\lambda(gz) &= \frac{g^3 z^2}{18\pi m_B} \frac{1}{(\lambda + c)^2} \left(\frac{2}{\beta} + \frac{1}{\alpha} \right)\end{aligned}\tag{7}$$

Finally, we will consider the Landau gauge where $z = 1$ and the differential equations that remain read then

$$\begin{aligned}\partial_\lambda \beta &= -N \frac{g^2}{6\pi m_B} \frac{1}{(\lambda + c)^2} \\ \partial_\lambda g &= \frac{g^3}{9\pi \beta m_B} \frac{1}{(\lambda + c)^2}.\end{aligned}\tag{8}$$

In this case, the dynamical mass is given by

$$m_{dyn} = c m_B.\tag{9}$$

We remind that the bare fermion mass is λm_B and not m_B , such that the massless case is reached when $\lambda \rightarrow 0$. m_B must be seen as an initial parameter which cannot vanish (the whole method is based on $m_B \neq 0$!). The possibility of having $m_{dyn} = 0$ then occurs if $c = 0$.

3 One-loop solution (massive case)

To obtain the renormalized parameters in the massive case, we integrate the differential equations (8) from $\lambda = \infty$ where the quantum corrections are frozen ($g(\infty) = g_B, \beta(\infty) = 1$), to $\lambda = 1$ where all the quantum effects are generated (we note $g(1) = g_1, \beta(1) = \beta_1$).

Before going to the full treatment of the differential equations (8), let us check that these lead to the expected results at one-loop. We note for this that the right-hand sides of Eqs.(8) are proportional to \hbar , such that the one-loop approximation consists in replacing the parameters of the right-hand sides by the bare ones (and thus take $c \rightarrow 0$ since this constant arises from quantum effects). We have then, restoring the factors \hbar ,

$$\begin{aligned}\partial_\lambda \beta &= -\hbar N \frac{g_B^2}{6\pi m_B} \frac{1}{\lambda^2} + \dots \\ \partial_\lambda g &= \hbar \frac{g_B^3}{9\pi m_B} \frac{1}{\lambda^2} + \dots,\end{aligned}\tag{10}$$

where the dots stand for higher orders in \hbar . The integration of these equations from $\lambda = \infty$ to $\lambda = 1$ gives then

$$\begin{aligned}\beta_1 &= 1 + \hbar N \frac{g_B^2}{6\pi m_B} + \dots \\ g_1 &= g_B - \hbar \frac{g_B^3}{9\pi m_B} + \dots\end{aligned}\tag{11}$$

It can be checked with standard computations that these results are given by the one-loop Feynman graphs. Indeed, the graph corresponding to the one-loop vacuum polarization (for one fermion flavour) is

$$\begin{aligned}& i(-ig_B)^2 \text{tr} \int_p \gamma^\mu \frac{\not{p} + m_B}{p^2 - m_B^2} \gamma^\nu \frac{(\not{p} - \not{k}) + m_B}{(p-k)^2 - m_B^2} \\ &= -i \frac{g_B^2 g^{\mu\nu}}{2\pi^2} \int_0^1 dx \int d^3 q \frac{m_B^2 - q^2 - x(1-x)k^2 + 2q^2/3}{[q^2 - m_B^2 + x(1-x)k^2]^2} \\ &+ \frac{4g_B^2}{\pi^2} k^2 T^{\mu\nu}(k) \int_0^1 dx \int_0^\infty dq_E \frac{x(1-x)q_E^2}{[q_E^2 + m_B^2 - x(1-x)k^2]^2}\end{aligned}\tag{12}$$

$$= \frac{g_B^2}{6\pi m_B} k^2 T^{\mu\nu}(k) + \text{higher orders in } k,$$

where $q = p - (1-x)k$ and q_E denotes the Euclidean momentum. As expected, the would-be linearly divergent integral (second line) cancels when using a gauge invariant regularization. The one-loop function β_1 for N flavours is then given by Eq.(11). Note that this result is consistent with the limit $k \ll m_B$ of the large- N polarization tensor [1].

The graph corresponding to the one-loop correction to the coupling is, in the Landau gauge and in the limit of zero incoming momenta,

$$\begin{aligned} & (-ig_B)^3 \int_p \frac{1}{p^2} T_{\mu\nu}(p) \gamma^\mu \frac{\not{p} + m_B}{p^2 - m_B^2} \gamma^\rho \frac{\not{p} + m_B}{p^2 - m_B^2} \gamma^\nu \\ &= -\frac{2g_B^3}{9} \gamma^\rho \int_{p_E} \frac{1}{p_E^2} \frac{p_E^2 + 3m_B^2}{(p_E^2 + m_B^2)^2} \\ &= -\frac{g_B^3}{9\pi m_B} \gamma^\rho, \end{aligned} \tag{13}$$

which leads to the result in Eq.(11).

4 Non-perturbative solution

We turn now to the complete resolution of the differential equations (8). Taking the ratio of these two equations and integrating, we have for any λ

$$\beta = \left(\frac{g_B}{g} \right)^{3N/2}. \tag{14}$$

Plugging this result in the differential equation (8) for g gives

$$g^{-3-3N/2} \partial_\lambda g = \frac{g_B^{-3N/2}}{9\pi m_B} \frac{1}{(\lambda + c)^2}. \tag{15}$$

The integration from $\lambda = \infty$ down to any λ leads to

$$g(\lambda) = g_B \left(1 + \frac{3N+4}{18\pi} \frac{g_B^2}{m(\lambda)} \right)^{-2/(3N+4)}. \tag{16}$$

The solution for the photon propagator renormalization is then

$$\beta(\lambda) = \left(\frac{g_B}{g(\lambda)} \right)^{3N/2} = \left(1 + \frac{3N+4}{18\pi} \frac{g_B^2}{m(\lambda)} \right)^{3N/(3N+4)}. \tag{17}$$

So as to discuss the dynamical mass generation, we should take the limit $\lambda \rightarrow 0$, where $m(0) = m_{dyn}$ and we note $g(0) = g_0$. We have then

$$g_0 = g_B \left(1 + \frac{3N+4}{18\pi} \frac{g_B^2}{m_{dyn}} \right)^{-2/(3N+4)} \quad (18)$$

From this last equation we see that the only possibility to have $m_{dyn} = 0$ is $g_0 = 0$. It is indeed expected that no dynamical mass can be generated in a non-interacting effective theory. Eq.(18) should be seen as a self-consistent relation between the renormalized parameters g_0 and m_{dyn} , relation which is parametrized by g_B . This result, valid in the present approximation of the gradient expansion, is independent of N .

5 Discussion

Let us first show the link between the equations obtained by this method and the Schwinger-Dyson equation for the fermion propagator.

One can see from the Appendix B that the evolution equation of the fermion propagator is

$$\begin{aligned} \partial_\lambda G^{-1}(k) &= -m_B - im_B g^2 z^2 \int_p \gamma_\mu G^2(p) \gamma_\nu D^{\mu\nu}(k+p) \\ &= -m_B - im_B g_B^2 \int_p \gamma_\mu G_B^2(p) \gamma_\nu D_B^{\mu\nu}(k+p) \\ &\quad + \text{higher orders in } \hbar, \end{aligned} \quad (19)$$

where G_B and $D_B^{\mu\nu}$ are the bare fermion and photon propagators respectively. But since $G_B^{-1}(k) = \not{k} - \lambda m_B$, one can note that

$$\partial_\lambda G_B(k) = m_B G_B^2(k), \quad (20)$$

such that the one-loop approximation to the evolution equation of G^{-1} can be written

$$\partial_\lambda G^{-1}(k) = \partial_\lambda G_B^{-1}(k) - ig_B^2 \int_p \gamma_\mu \partial_\lambda G_B(p) \gamma_\nu D_B^{\mu\nu}(k+p). \quad (21)$$

Since $D_B^{\mu\nu}$ is independent of λ , the previous equation integrates as

$$G^{-1}(k) = G_B^{-1}(k) - ig_B^2 \int_p \gamma_\mu G_B(p) \gamma_\nu D_B^{\mu\nu}(k+p). \quad (22)$$

Therefore the one-loop approximation to the evolution equation of the fermion propagator gives the usual one-loop self energy, which is also the one-loop truncation of the Schwinger-Dyson equation for the fermion propagator. The higher-order terms in \hbar are different though, since the resummations provided by the two methods are not the same.

The study of dynamical mass generation with the Schwinger-Dyson method [1, 2, 3] takes into account the momentum dependence of the fermion and photon self energies, what is not the case with the gradient expansion truncated to the present IR approximation. For this reason, the influence of the number of flavours is not discussed here. Indeed, it is the momentum-dependence of the fermion self energy which leads to a condition on the number of flavours: it was found in [2] that there is no dynamical mass generated above $N_c = 32/\pi^2$, and the studies made in [3] agree with the existence of N_c , and of the same order.

The next step thus, to discuss the influence of the number of flavours, would be to go further in the gradient expansion, so as to take into account the momentum dependence of the self energies. But this would lead to "partial-integro-differential" equations (derivative with respect to λ and integration with respect to the momentum) and would involve new approximations, similar to the ones used in the study of Schwinger-Dyson equations. The present scheme is based on the gradient expansion approximation *only* and is therefore believed to give a relevant insight into this problem of dynamical mass generation, besides the study of the role of N .

To obtain an explicit expression of the dynamical mass as a function of the bare coupling only (and therefore compute the value of the constant c in Eq.(9)), one would need to take a finite initial value for λ , corresponding to a system where some quantum fluctuations are already generated. The initial values of the parameters defining the theory could then be taken from a loop-expansion, valid when $\lambda \gg 1$. But the result would then depend on the initial value of λ , whereas the motivation of the present work was to find a self-consistent equation like (18), independent of the λ -scheme used here.

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Appendix A: Evolution equation for Γ

The bare action, functional of the fields $\hat{\bar{\psi}}, \hat{\psi}, \hat{A}_\mu$ is

$$S = \int d^3x \left\{ \frac{1}{2} \hat{A}^\mu (T_{\mu\nu} + \alpha L_{\mu\nu}) \hat{A}^\nu + \sum_{k=1}^N \hat{\bar{\psi}}_k (i \not{\partial} - g_B \not{\hat{A}} - \lambda m_B) \hat{\psi}_k \right\}, \quad (23)$$

and the connected graphs generator functional, function of the sources $\eta_k, \bar{\eta}_l, j_\mu$ is defined by $W = -i \ln \mathcal{Z}$ where

$$\mathcal{Z} = \int \mathcal{D}[\hat{\bar{\psi}}, \hat{\psi}, \hat{A}_\mu] \exp \left\{ iS + i \int d^3x \left(j^\mu \hat{A}_\mu + \sum_{k=1}^N (\hat{\bar{\psi}}_k \eta_k + \bar{\eta}_k \hat{\psi}_k) \right) \right\}. \quad (24)$$

The functional derivatives of W define the expectation value fields $\bar{\psi}_k, \psi_l, A_\mu$:

$$\begin{aligned} \frac{\delta W}{\delta \bar{\eta}_l} &= \frac{1}{\mathcal{Z}} \langle \hat{\psi}_l \rangle = \psi_l \\ \frac{\delta W}{\delta \eta_k} &= -\frac{1}{\mathcal{Z}} \langle \hat{\bar{\psi}}_k \rangle = -\bar{\psi}_k \\ \frac{\delta W}{\delta j_\mu} &= \frac{1}{\mathcal{Z}} \langle \hat{A}_\mu \rangle = A_\mu, \end{aligned} \quad (25)$$

where for any $\hat{\phi}$

$$\langle \hat{\phi} \rangle = \int \mathcal{D}[\hat{\phi}] \hat{\phi} \exp \left\{ iS + i \int d^3x \left(j^\mu \hat{A}_\mu + \sum_{k=1}^N (\hat{\bar{\psi}}_k \eta_k + \bar{\eta}_k \hat{\psi}_k) \right) \right\}. \quad (26)$$

We also have

$$\frac{\delta^2 W}{\delta \bar{\eta}_k \delta \eta_l} = i \bar{\psi}_l \psi_k - \frac{i}{\mathcal{Z}} \langle \hat{\bar{\psi}}_l \hat{\psi}_k \rangle. \quad (27)$$

Inverting the relations (25) which give the expectation value fields as functions of the sources, we define the Legendre transform Γ of W by

$$\Gamma = W - \int d^3x \left\{ j^\mu A_\mu + \sum_{k=1}^N (\bar{\psi}_k \eta_k + \bar{\eta}_k \psi_k) \right\}. \quad (28)$$

From this definition we extract the following functional derivatives:

$$\frac{\delta \Gamma}{\delta \psi_l} = \bar{\eta}_l \quad (29)$$

$$\frac{\delta \Gamma}{\delta \bar{\psi}_k} = -\eta_k \quad (30)$$

$$\frac{\delta \Gamma}{\delta A^\mu} = -j_\mu \quad (31)$$

$$\frac{\delta^2 \Gamma}{\delta \bar{\psi}_k \delta \psi_l} = - \left(\frac{\delta^2 W}{\delta \bar{\eta}_l \delta \eta_k} \right)^{-1}$$

The evolution of W with the parameter λ is, according to (27),

$$\begin{aligned}
\partial_\lambda W &= -\frac{m_B}{\mathcal{Z}} \int d^3x \sum_{k=1}^N \langle \hat{\bar{\psi}}_k \hat{\psi}_k \rangle \\
&= -m_B \int d^3x \sum_{k=1}^N \bar{\psi}_k \psi_k - im_B \int d^3x \sum_{k=1}^N \frac{\delta^2 W}{\delta \bar{\eta}_k \eta_k}.
\end{aligned} \tag{32}$$

To compute the evolution of Γ with λ , we have to keep in mind that its independent variables are $\bar{\psi}_k, \psi_k, A_\mu$ and λ , such that

$$\begin{aligned}
\partial_\lambda \Gamma &= \partial_\lambda W + \int d^3x \left\{ \frac{\delta W}{\delta j_\mu} \partial_\lambda j_\mu + \sum_{k=1}^N \left(-\frac{\delta W}{\delta \eta_k} \partial_\lambda \eta_k + \partial_\lambda \bar{\eta}_k \frac{\delta W}{\delta \bar{\eta}_k} \right) \right\} \\
&\quad - \int d^3x \left\{ \partial_\lambda j^\mu A_\mu + \sum_{k=1}^N (\bar{\psi}_k \partial_\lambda \eta_k + \partial_\lambda \bar{\eta}_k \psi_k) \right\} \\
&= \partial_\lambda W.
\end{aligned} \tag{33}$$

Combining these different results, we finally obtain the exact evolution equation for the proper graphs generator functional Γ :

$$\begin{aligned}
\partial_\lambda \Gamma &= m_B \int d^3x \sum_{k=1}^N \left\{ -\bar{\psi}_k \psi_k + i \left(\frac{\delta^2 \Gamma}{\delta \bar{\psi}_k \delta \psi_k} \right)^{-1} \right\} \\
&= m_B \int_{pq} \delta^3(p+q) \sum_{k=1}^N \left\{ -\bar{\psi}_k(p) \psi_k(q) + i \left(\frac{\delta^2 \Gamma}{\delta \bar{\psi}_k(p) \delta \psi_k(q)} \right)^{-1} \right\},
\end{aligned} \tag{34}$$

where we denote

$$\int_p(\dots) = \int \frac{d^3p}{(2\pi)^3}(\dots). \tag{35}$$

Appendix B: Evolution equations for the parameters

We decompose the second derivative of Γ in a sum of the field-independent diagonal part Δ and the part Θ which contains the fields. The inverse of $\delta^2 \Gamma$ is then taken as the following expansion

$$\begin{aligned}
(\delta^2 \Gamma)^{-1} &= \Delta^{-1} - \Delta^{-1} \Theta \Delta^{-1} + \Delta^{-1} \Theta \Delta^{-1} \Theta \Delta^{-1} \\
&\quad - \Delta^{-1} \Theta \Delta^{-1} \Theta \Delta^{-1} \Theta \Delta^{-1} + \dots,
\end{aligned} \tag{36}$$

and this expansion is truncated up to the desired order consistent with the approximation (3) that we consider, i.e. up to the order 3 in the non-diagonal part Θ . The result is then (the flavor indices are not written)

$$\text{Tr} \left(\frac{\delta^2 \Gamma}{\delta \bar{\psi} \delta \psi} \right)^{-1} = \text{Tr} \left\{ G + G \left(-\phi + \bar{\theta}^\mu D_{\mu\nu} \theta^\nu + \phi G \phi - 2\bar{\theta}^\mu D_{\mu\nu} \theta^\nu G \phi - \phi G \phi G \phi + \dots \right) G \right\}, \quad (37)$$

where (in Fourier components)

$$\begin{aligned} G(p, q) &= \frac{z \not{p} + m}{z^2 p^2 - m^2} \delta^3(p + q) \\ D_{\mu\nu}(p, q) &= \frac{1}{p^2} \left(\frac{1}{\beta_T} T_{\mu\nu}(p) + \frac{1}{\alpha \beta_L} L_{\mu\nu}(p) \right) \delta^3(p + q) \\ \phi(p, q) &= -zg \not{A}(-p - q) \\ \theta^\mu(p, q) &= -zg \gamma^\mu \psi(-p - q) \\ \bar{\theta}^\nu(p, q) &= zg \bar{\psi}(-p - q) \gamma^\nu, \end{aligned} \quad (38)$$

with

$$\begin{aligned} T_{\mu\nu}(p) &= g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \\ L_{\mu\nu}(p) &= \frac{p_\mu p_\nu}{p^2}. \end{aligned} \quad (39)$$

The contribution of Eq.(37) to the fermion propagator is then

$$\begin{aligned} &\text{Tr} \left\{ G \bar{\theta}^\mu D_{\mu\nu} \theta^\nu G \right\} \\ &= -g^2 z^2 \int_k \bar{\psi}(k) \left(\int_p \gamma^\nu G^2(p) \gamma^\mu D_{\mu\nu}(k + p) \right) \psi(-k) \\ &= -i \frac{g^2 z^2}{6\pi m^2} \frac{1}{\alpha \beta_L} \int_k \bar{\psi}(k) \not{k} \psi(-k) + \text{higher orders in } k, \end{aligned} \quad (40)$$

where the higher orders in k are not considered in the present approximation (3) of the gradient expansion. We note that the mass term (order k^0) vanishes. The contribution of Eq.(37) to the photon propagator is

$$\text{Tr} \{ G \phi G \phi G \} \quad (41)$$

$$\begin{aligned}
&= -g^2 z^2 \int_k A_\mu(-k) \left(\text{tr} \int_p G^2(p) \gamma^\mu G(k+p) \gamma^\nu \right) A_\nu(k) \\
&= i \frac{g^2 z^2}{12\pi m^2 z} \int_k A_\mu(-k) A_\nu(k) k^2 T^{\mu\nu}(k) + \text{higher orders in } k.
\end{aligned}$$

As expected, no photon mass is generated and the correction to the photon propagator is transverse. Finally, the contribution of Eq.(37) to the vertex is

$$\begin{aligned}
&\text{Tr} \left\{ -2G\bar{\theta}^\mu D_{\mu\nu} \theta^\nu G\phi G \right\} \tag{42} \\
&= -2g^3 z^3 \int_{kp} \bar{\psi}(k) A_\rho(-p) \left(\int_q \gamma^\mu G(q) \gamma^\rho G^2(p+q) \gamma^\nu D_{\mu\nu}(k+q) \right) \psi(p-k) \\
&= i \frac{g^3 z^2}{18\pi m^2} \left(\frac{2}{\beta_T} + \frac{1}{\alpha\beta_L} \right) \int_{kp} \bar{\psi}(k) \not{A}(-p) \psi(p-k) \\
&\quad + \text{higher orders in } k \text{ and } p.
\end{aligned}$$

The last contribution $G\phi G\phi G\phi G$ in Eq.(37) gives a cubic term in the photon field and vanishes by symmetry. The identification of the right-hand side and left hand side of the evolution equation (2) leads then to the differential equations (4).

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